B.Sc. Maths (HONS) I
Differential Calculus
Video Lecture Note: 1

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Successive Differentiation and Leibnitz's Theorem:

Definition & Notations:- Let y be a function of x and denoted by $y=f(x)$. Then differentiation of y w.r.t. x is called as Ist derivative of y or Ist differential coeff^{ts} of y or Ist derived function of y w.r.t. x and denoted by y_1 or y' or Dy or $\frac{dy}{dx}$ or $f'(x)$ or $f_1(x)$ or $\frac{d}{dx}f(x)$ or $Df(x)$. Similarly; second times differentiation of y w.r.t. x is called as IInd derivative or IInd differential coefficients or IInd derived function of y and denoted by y_2 or y'' or D^2y or $\frac{d^2y}{dx^2}$ or $D^2f(x)$ or $D(Df(x))$ or $f''(x)$ or $f_2(x)$ or $\frac{d^2f(x)}{dx^2}$.

Continuing same process; n times differentiation of y w.r.t. x is called as nth differential coeff^t or nth derivative or nth derived function of y w.r.t. x. and denoted by D^ny or $D(D^{n-1}y)$ or $\frac{d^ny}{dx^n}$ or y^n or y_n or $\frac{d^nf(x)}{dx^n}$ or $D^nf(x)$ or $f^n(x)$ etc.

The process of differentiating of the same function again and again is known as successive differentiation.

The derivatives obtained during this process are called successive derivatives.

Example:- Let $y=x^n$; then $\frac{dy}{dx}=nx^{n-1}$; $\frac{d^2y}{dx^2}=n(n-1)x^{n-2}$ ...... so and so on, we have $\frac{d^ny}{dx^n}=n(n-1)\cdots 2\cdot 1=\lfloor n$

Leibnitz's theorem:- If u and v are two functions of x and $y = uv$, the product of two function; then nth derivative of y is given by

$$y_n = u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \cdots + {}^nC_r u_{n-r} v_r + \cdots + u v_n.$$

This theorem is statement of Leibnitz's theorem.

Proof:- We have $y = uv$ ——— (I)

Differentiating w.r.t x; we have

$$y_1 = u_1 v + u v_1 \text{ ——— (II)}$$

which is true for $n=1$.

Again differentiating (II) w.r.t. x; we have

$$y_2 = u_2 v + u_1 v_1 + u_1 v_1 + u v_2 = u_2 v + 2u_1 v_1 + u v_2 \text{ ——— (III)}$$

which is true for $n = 2$.

Let us consider it is true for $n = m$, then we have.

$$y_m = u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \cdots + {}^mC_r u_{m-r} v_r + \cdots + u v_m \text{ ——— (IV)}$$

Differentiating w.r.t. x; we have.

$$y_{m+1} = u_{m+1} v + u_m v_1 + {}^mC_1 u_m v_1 + {}^mC_1 u_{m-1} v_2 + \cdots + {}^mC_r u_{m-r+1} v_r + {}^mC_r u_{m-r} v_{r+1} + \cdots + {}^mC_m u_1 v_m + u v_{m+1}$$

Using Pascal's law ${}^mC_{r-1} + {}^mC_r = {}^{m+1}C_r$ & arranging the terms; we have

$$y_{m+1} = u_{m+1} v + {}^{m+1}C_1 u_m v_1 + {}^{m+1}C_2 u_{m-1} v_2 + \cdots + u v_{m+1} \text{ ——— (V)}$$

$\Rightarrow$ It is also true for $n = m+1$.

Hence it is true for all values of n. (By Mathematical Induction method)

Hence the Leibnitz's theorem proved.