B.Sc. Ist Year
Paper- IInd
Lecture Note- 2
(Differential Calculus)

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Successive Differentiation

Calculations of nth derivative of some standard functions :

(i) Let $y = (ax+b)^m$

$\Rightarrow y_1 = m.(ax+b)^{m-1}.a.1$ $\left(\frac{d x^k}{dx} = k x^{k-1}\right)$

$\Rightarrow y_2 = m.(m-1)(ax+b)^{m-2}.a^2$

$\Rightarrow y_3 = m(m-1)(m-2)(ax+b)^{m-3}.a^3$

Similarly, we have

$$y_n = m(m-1)(m-2)\cdots(m-n+1)(ax+b)^{m-n}.a^n ; n \leq m$$

$$y_n = \frac{m(m-1)(m-2)\cdots(m-n+1)(m-n)\cdots 2.1}{(m-n)\cdots 2.1}.a^n (ax+b)^{m-n}$$

$$y_n = \frac{\lfloor m}{\lfloor m-n}.a^n (ax+b)^{m-n}$$

which is required nth derivative.

(ii) Let $y = e^{ax+b}$

$\Rightarrow y_1 = e^{ax+b}.a$ $\left(\frac{d e^x}{dx} = e^x\right)$

$\Rightarrow y_2 = e^{ax+b}.a^2$

$\Rightarrow y_3 = e^{ax+b}.a^3$

Similarly, we obtained

$$y_n = e^{ax+b}.a^n$$

$$\Rightarrow y_n = a^n.e^{ax+b}$$

which is required nth derivative.

(iii) Let $y = \sin(ax+b)$

$\Rightarrow y_1 = \cos(ax+b) \cdot a$ $\quad \left(\frac{d}{dx}\sin x = \cos x\right)$

$\Rightarrow y_1 = a\sin\left(ax+b+\frac{\pi}{2}\right)$ $\quad \left(\text{As } \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta\right)$

$\Rightarrow y_2 = a\cos\left(ax+b+\frac{\pi}{2}\right) \cdot a$

$\Rightarrow y_2 = a^2 \sin\left(ax+b+\frac{2\pi}{2}\right)$

Similarly; we obtained

$$y_n = a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$$

Which is required nth derivative.

(iv) Let $y = a^{mx}$

$\Rightarrow y_1 = a^{mx} \cdot \log_e(a) \cdot m$ $\quad \left(\frac{d}{dx} a^x = a^x \log_e a\right)$

$\Rightarrow y_2 = a^{mx} \cdot \log_e a \cdot m \cdot \log_e(a) \cdot m$

$\Rightarrow y_2 = m^2 (\log_e a)^2 \cdot a^{mx}$

Similarly; we obtained

$$y_n = m^n (\log_e a)^n \cdot a^{mx}$$

Which is required nth derivative.

Solve:

Exercise: If $y = (x^2-1)^n$; prove that $y_{2n} = \lfloor(2n)$.

Solution: Given that $y = (x^2-1)^n$

By Binomial expansion; we have

$$y = x^{2n} - n \cdot x^{2n-2} + \frac{n(n-1)}{\lfloor 2} x^{2n-4} - \cdots$$

Differentiating $2n$ times w.r.t. x; we have

$$y_{2n} = \lfloor(2n) \cdot 1 - 0 + 0 - \cdots$$

$\Rightarrow y_{2n} = \lfloor(2n)$. (Hence proved).