B.Sc. Ist Year Maths Hons. & B.Sc. IInd Year Maths Subs. Differential Calculus (Paper- IInd).

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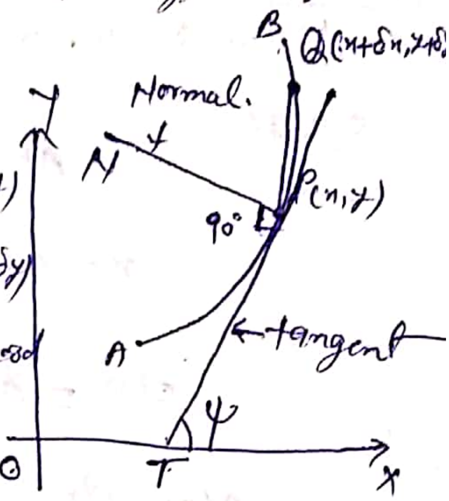
Tangents & Normals :-



Let APB be a given curve and $P(x,y)$ be a point on curve. Let $Q(x+\delta x, y+\delta y)$ another point on curve in neighbourhood of P. As Q tends to P; the chord PQ tents to a definite line PT, which is called as tangents at point P on the curve APB.

A line PN is said to be normal at the point P on the curve APB; which passess through from the point P and makes a right angle i.e. $\perp^{ar}$ to the tangent line PT.

Equation of Tangents and Normals :-

The eqn of line passing through the points P and Q is given by

$$Y-y=\left(\frac{y+\delta y-y}{x+\delta x-x}\right)(X-x) \qquad \left(y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)\right)$$

$$\Rightarrow Y-y=\left(\frac{\delta y}{\delta x}\right)(X-x) \quad \text{——— (I)}$$

as Q tends to P; $\frac{\delta y}{\delta x}$ reduces into $\frac{dy}{dx}$.

$$\Rightarrow Y-y=\left(\frac{dy}{dx}\right)(X-x), \text{ as } Q\to P. \quad \text{——— (II)}$$

$$\Rightarrow Y = y + \frac{dy}{dx}(X - x)$$

$$\Rightarrow Y = \left(\frac{dy}{dx}\right)X + \left(y - x\frac{dy}{dx}\right) \quad \text{--- (iii)}$$

In general eqn of straight line is given by

$Y = mX + c$ in gradient-intersection form. — (iv)

where $m = \tan\varphi$; φ is an angle made by line from +ve direction of x-axis, anticlockwise.

Comparing eqn (iii) & (iv); we have

$$m = \tan\varphi = \frac{dy}{dx} \quad \text{--- (v)}$$

Let m' be the gradient of the normal line.

So since tangents and normal are $\perp^{ar}$.

$$\Rightarrow m \cdot m' = -1 \Rightarrow m' = -\frac{1}{m}$$

$$= -\frac{dx}{dy} \quad (\text{using (v)})$$

So eqn of normal at the point P is given by $Y = m'X + \left(y + x\frac{dx}{dy}\right)$.

i.e. $Y - y = \left(-\frac{dx}{dy}\right)(X - x)$

$\Rightarrow \boxed{Y = m'X + C'}$ is eqn of normal PN.

where $m' = -\frac{dx}{dy}$; $c' = y + x\frac{dx}{dy}$.

Let eqn of the curve given in form of parametric eqn $x = \phi(t)$; $y = g(t)$ (say).

$$\frac{dx}{dt} = \phi'(t) \; ; \; \frac{dy}{dt} = g'(t)$$

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{g'(t)}{\phi'(t)}.$$

$\Rightarrow$ eqn of tangent is given by

$$Y - g(t) = \left\{\frac{g'(t)}{\phi'(t)}\right\}\left\{X - \phi(t)\right\}.$$

& eqn of Normal is given by.

$$Y - g(t) = \left\{-\frac{\phi'(t)}{g'(t)}\right\}\left\{X - \phi(t)\right\}.$$

<u>Case I</u>: Let tangent is parallel to x-axis.

$$\frac{dy}{dx} = 0 \quad \Rightarrow \text{eqn of tangent } Y - y = 0.$$

& eqn of normal $(Y - y) = \left(-\frac{dx}{dy}\right)(X - x)$

$$\Rightarrow (Y - y) = -\frac{1}{0}(X - x)$$

$$\Rightarrow X - x = 0$$

<u>Case II</u>: Similarly if tangent is parallel to y-axis; eqn of tangent is $X - x = 0$ and eqn of normal is $Y - y = 0$.

Example 8 :- Find the eqn of tangent and normal at $\theta = \pi/2$ to the curve. $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

Solution :- At $\theta = \pi/2$

$$x = a\left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right) ; \; y = a(1 + \cos \pi/2)$$

$$\Rightarrow x = a\left(\frac{\pi}{2} + 1\right) ; \; y = a \; ; \text{ at } \theta = \pi/2$$

Also $\frac{dx}{d\theta} = a(1 + \cos\theta)$; $\frac{dy}{d\theta} = a(-\sin\theta)$

at $\theta = \pi/2$; $\frac{dx}{d\theta} = a$; $\frac{dy}{d\theta} = -a$.

so $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{a}{a} = -1$; at $\theta = \pi/2$

so eqn of tangent at $\theta = \pi/2$ is given by

$$Y - y = \left(\frac{dy}{dx}\right)(X - x) \; ; \text{ at } \theta = \pi/2$$

$$\Rightarrow Y - a = (-1)\left(X - a\left(\frac{\pi}{2} + 1\right)\right)$$

$$\Rightarrow Y = a - X + a \cdot \frac{\pi}{2} + a$$

$$\Rightarrow X + Y = a\left(2 + \frac{\pi}{2}\right)$$

required eqn of tangent

And eqn of normal at $\theta = \pi/2$ is given by

$$Y - y = -\frac{1}{(-1)}(X - x) \; ; \text{ at } \theta = \pi/2$$

$$\Rightarrow Y - a = \left(X - a\left(\frac{\pi}{2} + 1\right)\right)$$

$$\Rightarrow Y - X + a \cdot \frac{\pi}{2} = 0$$

required eqn of normal.