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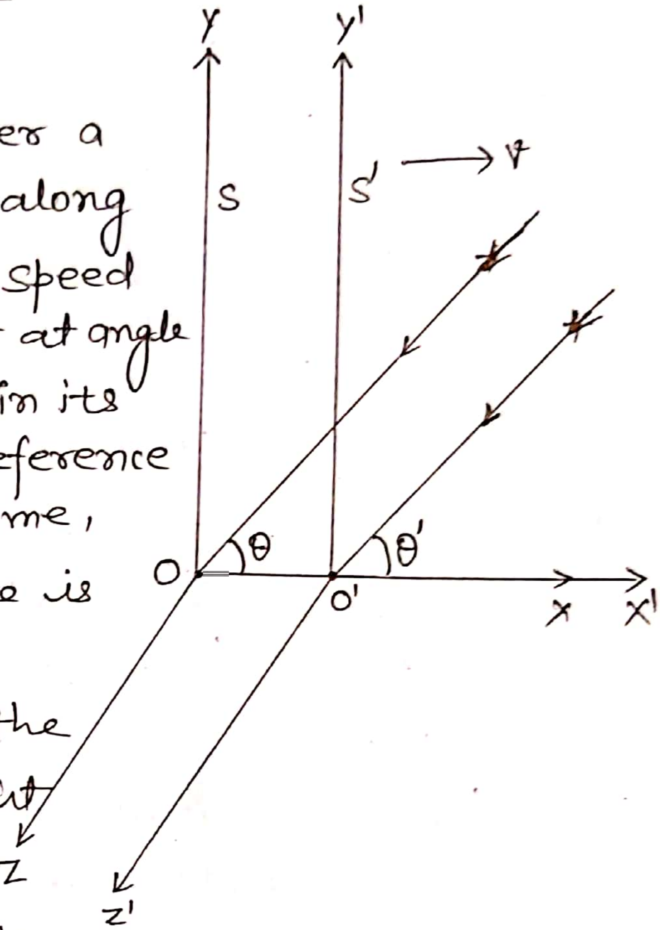
B.Sc (Part-I) Hons

Aberation of light on the theory of relativity :—



Let us Consider a Source s' moving along the x-axis at a speed of v emit light at angle θ' to the x'-axis in its own frame of reference s'. In the s-frame, the emit angle is θ. The velocity Components of the velocity of light c in the x'-direction is $u_x = c\cos\theta'$ and in the y'-direction it is $u'_y = c\sin\theta'$

$$\therefore \frac{u'_y}{u'_x} = \tan\theta$$

$$\therefore \tan\theta = \frac{u'_y}{u'_x} \quad \text{———} \quad (1)$$

Using the formula for the relativistic addition of velocities,

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$$

$$u_x = \frac{c\cos\theta' + v}{1 + \frac{v\cos\theta'}{c}}$$

and
$$u_y = \frac{u'_y\sqrt{1 - v^2/c^2}}{1 + u'_x v/c^2}$$

$$u_y = \frac{c\sin\theta'\sqrt{1 - v^2/c^2}}{1 + \frac{c\cos\theta' . v}{c^2}}$$

$$\therefore u_y = \frac{c\sin\theta'\sqrt{1 - v^2/c^2}}{1 + \frac{v\cos\theta'}{c}}$$

$$\therefore \tan\theta = \frac{u_y}{u_x}$$

$$= \frac{c\sin\theta'\sqrt{1 - v^2/c^2}}{c\cos\theta' + v}$$

$$= \frac{c\sin\theta'\sqrt{1 - v^2/c^2}}{c(\cos\theta' + v/c)}$$

$$\therefore \tan\theta = \frac{\sin\theta'\sqrt{1 - v^2/c^2}}{\cos\theta' + v/c} \quad \text{--- (2)}$$

This is the relativistic aberration formula. The inverse transformation can at once be written by putting $-v$ for v and interchanging primed and unprimed quantities.

Hence
$$\tan\theta' = \frac{\sin\theta\sqrt{1 - v^2/c^2}}{\cos\theta - v/c} \quad \text{--- (3)}$$

Now, let us consider the case of a star directly overhead in the S-frame. One receives plane waves whose direction of propagation is along the negative direction of y-axis.

Hence

$$\theta = \frac{3\pi}{2}$$

In the frame s', the propagation direction is θ' given by eqn ③

$$\therefore \tan\theta' = \frac{\sin\frac{3\pi}{2}\sqrt{1-v^2/c^2}}{\cos\frac{3\pi}{2} - \frac{v}{c}}$$

$$= \frac{-\sqrt{1-v^2/c^2}}{-v/c}$$

$$= \frac{\sqrt{1-v^2/c^2}}{v/c}$$

$$\therefore \tan\theta' = \frac{1-\frac{1}{2}\frac{v^2}{c^2}}{v/c}$$

Since $v \ll c$, v^2/c^2 can be neglected

$$\therefore \tan\theta' = \frac{1}{v/c}$$

$$\therefore \boxed{\tan\theta' = \frac{c}{v}} \quad \text{———} \quad (4)$$

The apparent change in angle from the star direction has been found to be

$$\alpha = \tan^{-1}\left(\frac{v}{c}\right)$$

$$= \tan^{-1}\left(\frac{30\,km/s}{3\times10^5\,km/s}\right)$$

$$= \tan^{-1}(10^{-4}) = 20.5 \text{ sec of arc}$$

which is in perfect agreement with the observed first order aberration effect corresponding to the classical interpretation of the situation.