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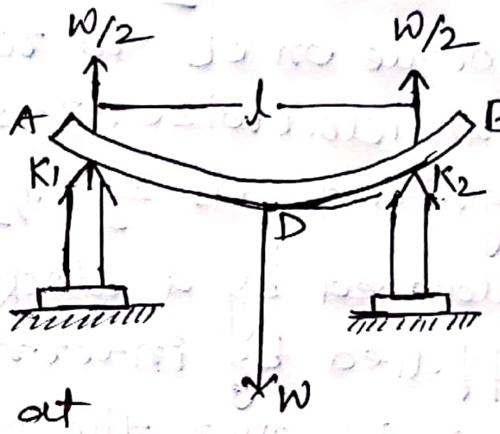
B.Sc (Part-1) Hons paper-1

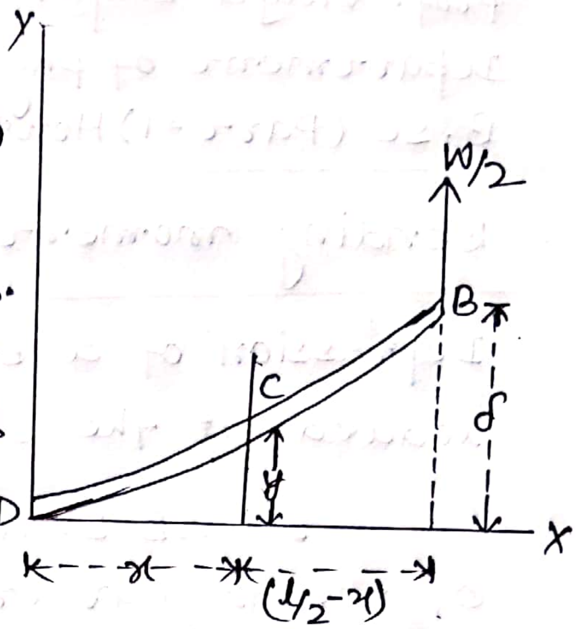
Bending moment of a metalic beam.

Depression of a beam supported at the ends and loaded at the Centre:-



Let AB be a beam of rectangular Cross-Section resting symmetrically on two horizontal Knife-edges K_1 and K_2 and is load at the middle point D by a weight W. The reaction at the Knife-edges is $\frac{W}{2}$ acting vertically upward. The beam bends symmetrically with respect to its middle point D at which the depression a maximum. Let l be the distance between the Knife-edges K_1 and K_2. On account of the symm-etrical bending on the two sides of D, a tangent drawn at D will be horizontal. Hence each half of the beam, DA and DB may be considered as Cantilever of length $l/2$ fixed at one end D and loaded by an upward load $\frac{W}{2}$ at the other. The elevation of the loaded end of any Cantilever will be the same as the depression in the middle in the equal Case. Therefore we have to simply determine the elevation of B above D.

Let us take a section at C at a distance x from D. Let us now consider the equilibrium of the part CB. Since the beam is fixed at D, the load $\frac{W}{2}$ at B exerts a torque on CB to rotate D in anticlockwise. Its



magnitude is $\frac{W}{2}(l/2 - x)$. This torque is balanced by a clockwise restoring couple supplied by internal forces exerted by the part DC over the section C. These forces arise due to the elastic reaction of the beam against the extension of filament as one side of the neutral surface and compression on the other. The magnitude of the restoring couple is $\frac{YI}{\rho}$, where the symbols have the usual meanings. At equilibrium.

$$\frac{W}{2}\left(\frac{l}{2} - x\right) = \frac{YI}{\rho}$$

Let y be the elevation at C and (x, y) be the co-ordinates of C. Then the curvature at C is given by

$$\frac{1}{\rho} = \frac{d^2y}{dx^2}$$

$$\therefore \; YI \cdot \frac{d^2y}{dx^2} = \frac{W}{2}\left(\frac{l}{2} - x\right)$$

$$\text{or,} \quad \frac{d^2y}{dx^2} = \frac{W}{2YI}\left(\frac{l}{2} - x\right)$$

or, $\frac{dy}{dx} = \frac{w}{2YI}\left(\frac{l}{2}\cdot x - \frac{x^2}{2}\right) + A_1$

where A_1 is a constant of integration

At D, $x = 0$ and $\frac{dy}{dx} = 0$

$\therefore A_1 = 0$

$$\therefore \frac{dy}{dx} = \frac{w}{2YI}\left(\frac{lx}{2} - \frac{x^2}{2}\right) \quad \text{——— (1)}$$

$$\therefore \int dy = \frac{w}{2YI}\int\left(\frac{lx}{2} - \frac{x^2}{2}\right)dx$$

$$\therefore y = \frac{w}{2YI}\left(\frac{lx^2}{4} - \frac{x^3}{6}\right) + A_2$$

Again at D, $x = 0$ and $y = 0$ $\therefore A_2 = 0$

$$\therefore y = \frac{w}{2YI}\left(\frac{lx^2}{4} - \frac{x^3}{6}\right) \quad \text{——— (2)}$$

At B, $x = \frac{l}{2}$ and the elevation $y = \delta$ (maximum)

$\therefore$ From equation (2), we have

$$\delta = \frac{w}{2YI}\left(\frac{l(l/2)^2}{4} - \frac{(l/2)^3}{6}\right)$$

$$\therefore \boxed{\delta = \frac{wl^3}{48YI}}$$

If b and d be the width and the thickness of the beam respectively, we have

$$I = \frac{bd^3}{12}$$

$$\therefore \boxed{\delta = \frac{wl^3}{4Ybd^3}}$$

This is required equation for the depression of the middle point D. If the weight of the beam is not negligible but W_0, then $\frac{5}{8}$ of the weight is to be added to w to get the required depression.

$$\boxed{\delta = \frac{(W + \frac{5}{8}W_0)\,l^3}{4Ybd^3}}$$