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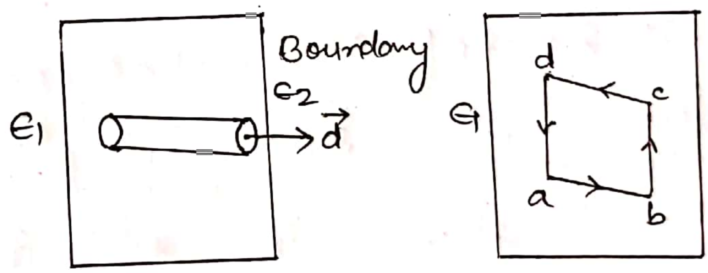
Topic :- Boundary Conditions for electric line of force.

<u>Boundary Conditions</u> :- Let us consider a Small pill-box as Gaussian Surface with its Curved Surface normal to the boundary and one flat-surface parallel to the boundary in the medium of permitivity ϵ_1 and the other parallel flat-surface in the medium of permitivity ϵ_2. Let us apply Gauss' law for $\vec{B}$ to the pil-Box. The net flux through the Gaussian Surface must be zero as it incloses no free charge.



The flux through the Curved Surface of cylinder may be neglected since the area can be made vanishingly Small by making the length of cylinder very Small as compared to its dimeter i.e by taking the end-face very close to the boundary. If we call the end face on the left-hand side S_1 and that as right-hand Side S_2, we may write

$$\int_S \vec{D}\cdot d\vec{S} = \int_{S_1} \vec{D}_1\cdot d\vec{S} + \int_{S_2} \vec{D}_2\cdot d\vec{S}$$

$$= \int_{S_1} D_{n_1}\, ds + \int_{S_2} D_{n_2}\, ds = 0$$

where $\vec{D}_1$ and $\vec{D}$ are the electric displacements in the media of permitivity ϵ_1 and ϵ_2 respectively and D_{n_1} and D_{n_2} are the normal Components of $\vec{D}_1$ and $\vec{D}_2$ respectively. The Sign of the integral as the left hand side of the boundary has been taken to be negative because the lines of $\vec{D}$ point in to Gaussian volume i.e the flux is in the direction of inward normal. It follows directly from the above equation that

$$D_{n_1} = D_{n_2} \qquad (1)$$

This show that the normal Component of D is Continous across the boundary.

Now let us Consider the closed path abcd with side ab and cd perpendicular to the boundary between the two media to Calculate the boundary Condition for $\vec{E}$ across the boundary of the dielectrics. The line integral of the electric field $\vec{E}$ round the closed path will be zero i.e.

$$\oint \vec{E}\cdot d\vec{l} = 0$$

The side ab and cd of the path Can be taken to be vanishingly Small so that the integral Can be expressed as

$$\oint \vec{E}\cdot\vec{dl} = \int_b^c \vec{E_2}\cdot\vec{dl} + \int_d^a \vec{E_1}\cdot\vec{dl}$$

$$= \int_b^c \vec{Et_2}\,\vec{dl} + \int_d^a \vec{Et_1}\cdot\vec{dl} = 0$$

$$= \int_b^c Et_2\,dl + \int_d^a Et_1\,\vec{dl} = 0 \qquad (2)$$

where $\vec{E_1}$ and $\vec{E_2}$ are the intensity of electric fields in the media of permittivity ϵ_1 and ϵ_2 respectively and Et_1 and Et_2 are the respective tangential Components

$$\because bc = da$$

$$\therefore Et_1 = Et_2 \qquad (3)$$

Thus the Components of E tangential to the boundary is continuous across the boundary.