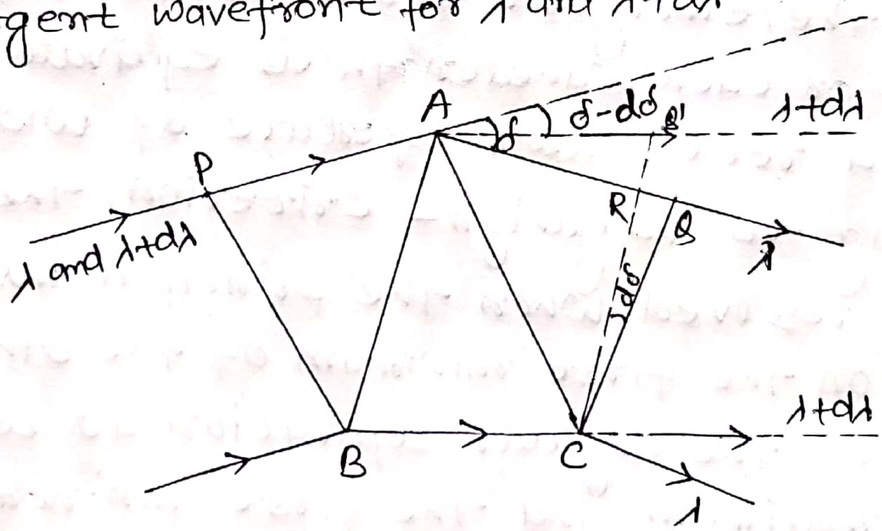
Prof. Vidya Sagar
Department of Physics
Raja Singh College, Siwan

B.Sc (Part-II) Hons
Paper-III

Chromatic Resolving power of prism

The chromatic resolving power of prism represents its ability to form seprate spectral lines for wavelengths very close together. It is measured by $\lambda/d\lambda$, where $d\lambda$ is the smallest wave-length difference that can be just resolved by the prism at the wave length λ.

Let ABC be the section of prism. Let a plane wavefront BP of light of wavelength λ and $\lambda+d\lambda$ be incident on the prism placed in the position on minimum deviation. Let CQ and CQ' be the emergent wavefront for λ and $\lambda+d\lambda$.



Let δ and $(\delta-d\delta)$ be the angles of deviation and μ and $(\mu-d\mu)$ the refractive indices corresponding to wavelength λ and $\lambda+d\lambda$ respectively. Clearly

$$\angle Q'CQ = d\delta$$

By Fermat's principal, the optical path between the incident and the emergent wavefront for any wavelength must be the same. Hence for the wavelength λ, we have

$$PA + AQ = \mu BC \quad \text{——} (1)$$

and for the wavelength $\lambda + d\lambda$, we have

$$PA + AQ' = (\mu - d\mu) BC \quad \text{——} (2)$$

subtracting (2) from (1) we get

$$AQ - AQ' = d\mu BC$$

But from figure $AQ' \simeq AR$, so that

$$AQ - AQ' = AQ - AR = RQ$$

$$\therefore RQ = d\mu BC$$

or, $CQ d\delta = d\mu BC$

If $CQ = e$ and $BC = t$ then

$$e d\delta = d\mu t$$

or, $$d\delta = \frac{t d\mu}{e} \quad \text{——} (3)$$

The emergent beam has rectangular cross-section. The prism may also be considered as rectangular aperture of width e which is the width of the emergent beam. Hence the diffraction pattern for each wavelength is equivalent to that due to a rectangular aperture of width e.

According to Rayleigh's criterion, the two lines are just resolved when the principal maximum of one fall on the first minimum of the other. This means that the angular separation $d\delta$ between the principal maximum and the first minimum of λ. The latter angle, from the results of Fraunhofer diffraction at a simple slit, is given by

$$e \sin d\theta = \lambda$$

or, $e d\theta = \lambda$ (as $d\theta$ is small, $\sin d\theta = d\theta$)

or, $$d\theta = \lambda / e \quad \text{——} (4)$$

For just resolution $d\delta = d\theta$. Hence from eqn (3) and (4), we get

$$\frac{t\,d\mu}{e} = \frac{\lambda}{e}$$

or, $t\,d\mu = \lambda$

Hence the resolving power of the prism

$$\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

This shows that the resolving power of a prism varies directly as the width of the base of the prism, and also as the rate of change of refractive index with wavelength.