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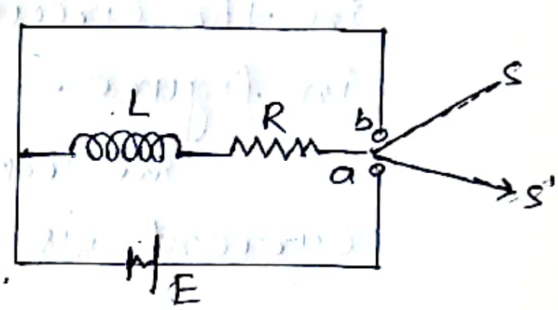
Decay of Current in L-R Circuit.

When the switch S is thrown over to b, the e.m.f. E applied to the circuit becomes zero, the resistance of the circuit remaining constant.



The self induction of the coil now opposes the fall of current. During the fall, let I be the current at any time t. In this case there is no source of e.m.f.

$$\therefore E = 0$$

induced e.m.f. across the coil $= -L\frac{dI}{dt}$

potential drop across the resistance $= IR$

$$\therefore -L\frac{dI}{dt} = IR$$

$$\text{or, } \frac{dI}{dt} = -\frac{IR}{L}$$

$$\therefore dt = -\frac{L}{R}\left(\frac{dI}{I}\right)$$

$$\text{or, } \int dt = -\frac{L}{R}\int\frac{dI}{I}$$

$$\therefore t = -\frac{L}{R}\log_e(I) + C \quad \text{———— (1)}$$

At $t=0$, $I = I_0$

$$\therefore C = \frac{L}{R}\log_e(I_0)$$

From eqn (1), we get

$$t = -\frac{L}{R}\log_e(I) + \frac{L}{R}\log_e(I_0)$$

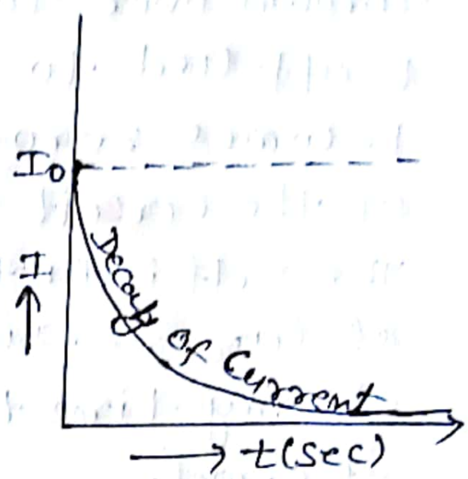
$= -\frac{L}{R} \log (I/I_0)$

$\therefore \log_e\left(\frac{I}{I_0}\right) = -\left(\frac{R}{L}\right)t$

$\therefore I = I_0 e^{-\left(\frac{R}{L}\right)t} = I_0 e^{-t/\lambda}$ ——— (2)

This equation shows that the current in the circuit decays exponentially shown in figure.

The rate of fall of current is



$\frac{dI}{dt} = -I_0 \frac{R}{L} e^{-\left(\frac{R}{L}\right)t}$

$\therefore \frac{dI}{dt} = -I_0\left(\frac{R}{L}\right)\frac{I}{I_0}$

$\therefore \frac{dI}{dt} = -\left(\frac{R}{L}\right)I$ ——— (3)

This is clear that greater the ratio R/L or smaller the time constant L/R, the current dies away more rapidly.

putting $t = \lambda$ in eqn(2) we have

$I = I_0 e^{-1} = I_0 \times \frac{1}{e}$ ——— (4)

or, $I = 0.37 I_0$

Thus the time constant $\lambda = \frac{L}{R}$ can be defined as the time taken by the current to fall $\frac{1}{e}$ or 37% of its maximum value.

Thus we see that the time constant is a measure of the rate at which a current grows or decays in an inductive current circuit.

When the source of emf is withdrawn from the circuit, the collapsing magnetic field in the induction coil becomes the source of energy for the decay of current.