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Derivation of the Newton's Law of Gravitation on the basic of Keplar's Law

Suppose that the orbit of planet is Circular. Let M_1, R_1, and T_1 be the mass, radius and period of revolution respectively of a planet.

Force exerted by the Sun on the planet is

$$F_1 = M_1 R_1 \omega_1^2$$

$$F_1 = M_1 R_1 \left(\frac{2\pi}{T_1}\right)^2 \quad \text{—— (I)}$$

Let M_2, R_2 and T_2 be the mass, radius and period of revolution respectively of a second planet.

Then the force exerted by the Sun on the second planet is

$$\vec{F_2} = M_2 R_2 \left(\frac{2\pi}{T_2}\right)^2 \quad \text{—— (II)}$$

$$\therefore \frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)\left(\frac{T_2}{T_1}\right)^2 \quad \text{—— (III)}$$

But from Keplar's law

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

from equation (iii)

$$\frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)\left(\frac{R_2}{R_1}\right)^3$$

or, $$\frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_2}{R_1}\right)^2$$

or, $$\frac{F_1 R_1^2}{M_1} = \frac{F_2 R_2^2}{M_2} = \text{Constant } (K)$$

then $F \propto \frac{M}{R^2}$

or, $F \propto M$ and $F \propto \frac{1}{R^2}$

Hence the force exerted by the Sun on the planet is proportional to the mass of the planet and inversely proportional to the square of distance from the Sun.

This is Newton's Law of Gravitation.