Prof. Vidya Sagar
Department of physics
B.Sc (Part-1) Hons

## Mass-Energy Relationship

According to Einstein's special theory of relativity, the mass $m$ of a body moving with velocity $v$ relative to a stationary observer varies with $v$ as

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \quad \text{--- (1)}$$

where $m_0$ is the rest mass and $c$ is the velocity of light. The varation of mass with velocity has modified our ideas about energy.

Let us consider a particle of mass $m$ acted upon by a force $F$ in the Same direction as its velocity $v$. The force is defined as the rate of change of momentum

$$\therefore F = \frac{d}{dt}(mv)$$

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{--- (2)}$$

The work done by the force is displacing the particle through a distance $dx$, is

$$dw = F \cdot dx$$

$$= \left[m\frac{dv}{dt} + v\frac{dm}{dt}\right]dx$$

$$= m dv \frac{dx}{dt} + v dm \frac{dx}{dt}$$

$$\therefore dw = mv dv + v^2 dm \quad \left(\because v = \frac{dx}{dt}\right)$$

This is equal to the change in the kinetic energy K of the body.

$$\therefore dK = F \cdot dx$$

$$\therefore dK = mv dv + v^2 dm \quad \text{——— (3)}$$

From equation (1)

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\therefore m^2 = \frac{m_0^2}{1 - v^2/c^2}$$

$$\text{or, } m^2 (1 - v^2/c^2) = m_0^2$$

$$\text{or, } m^2 \left(\frac{c^2 - v^2}{c^2}\right) = m_0^2$$

$$\text{or, } m^2 (c^2 - v^2) = m_0^2 c^2$$

$$\text{or, } m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

$$\text{or, } 2m dm . c^2 - (2m dm \times v^2 + m^2 \times 2v dv) = 0$$

$$\text{or, } 2m [c^2 dm - v^2 dm - mv dv] = 0$$

$$\text{or, } c^2 dm - v^2 dm - mv dv = 0$$

$$\therefore mv dv = c^2 dm - v^2 dm$$

putting this value in equation (3)

$$dK = c^2 dm - v^2 dm + v^2 dm$$

$$\therefore dK = c^2 dm \quad \text{——— (4)}$$

Let us suppose that the body has a rest mass $m_0$ when at rest and a mass $m$ when accelerated to a velocity v. then the K.E is

$$K = \int dK$$

$$= \int_{m_0}^{m} c^2 dm$$

$$= c^2 [m]_{m_0}^{m}$$

$$\therefore K = c^2(m - m_0) \quad \text{--- (5)}$$

The result show that the kinetic energy of a body is equal to the relativistic increase in mass of the body over the rest mass multiplied by the velocity of light. This it is concluded that even when the body is at rest, it passes an amount of energy $m_0c^2$. This energy is called the rest energy $E_0$.

$$\therefore E_0 = m_0c^2 \quad \text{--- (6)}$$

Then total energy

$$E = K + E_0$$

$$= c^2(m - m_0) + m_0c^2$$

$$= mc^2 - m_0c^2 + m_0c^2$$

$$\therefore E = mc^2 \quad \text{--- (7)}$$

This is the famous Einstein's mass-energy relation