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Fourier's Theorem

Any Continuous Single-valued periodic function can be expressed as a Summation of Simple harmonic terms having frequencies which are multiple of that of the given function.

Thus, any Complex Sound of frequency n can be analysed in to number of pure tones of frequencies $n, 2n, 3n,$ ----- and appropriate amplitudes.

Mathematically, The Fourier's theorem can be expressed as,

$$y = f(\omega t) = A_0 + A_1 \cos\omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + \cdots$$
$$\cdots A_r \cos r\omega t + B_1 \sin\omega t + B_2 \sin 2\omega t$$
$$+ B_3 \sin 3\omega t + \cdots + B_r \sin r\omega t \quad \text{--- (1)}$$

where y is the displacement of a Complex periodic motion of frequency $\omega/2\pi$. Thus the complex motion is the Sum of Sine and Cosine components of amplitudes $A_1, A_2, A_3, \cdots B_1, B_2, B_3 \cdots$ and frequencies which are multiples of $\omega/2\pi$. A_0 is a Constant representing the displacement of the axis of vibration Curve from the axis of co-ordinates.

In order to use this theorem for analysing a Complex wave, we have to evaluate the Constant A_0, A_r and B_r.

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Evaluation of A_0 :— Let us multiply equation (1) by dt and integrate from 0 to T.

$$\therefore \int_0^T y\,dt = A_0\int_0^T dt + A_1\int_0^T \cos\omega t\,dt + \cdots + A_r\int_0^T \cos r\omega t\,dt$$
$$+ B_1\int_0^T \sin\omega t\,dt + \cdots + B_r\int_0^T \sin r\omega t\,dt$$
$$= A_0 T \quad \text{(because other term are zero)}$$
$$\therefore A_0 = \frac{1}{T}\int_0^T y\,dt \qquad \text{——— (2)}$$

Evaluation of A_r :— Let us multiply equation (1) by $\cos r\omega t\,dt$ and integrate from 0 to T.

$$\therefore \int_0^T y\cos r\omega t = A_0\int_0^T \cos r\omega t\,dt + A_1\int_0^T \cos\omega t.\cos r\omega t\,dt \cdots$$
$$+ A_r\int_0^T \cos^2 r\omega t\,dt + B_1\int_0^T \sin\omega t.\cos r\omega t\,dt +$$
$$\cdots + B_r\int_0^T \sin r\omega t.\cos r\omega t\,dt$$
$$= A_r\int_0^T \cos^2 r\omega t\,dt, \quad \text{(the other term is zero)}$$
$$\therefore \int_0^T y\cos r\omega t\,dt = A_r\int_0^T \left(\frac{1+\cos 2r\omega t}{2}\right)dt$$
$$= \frac{A_r}{2}\left[\int_0^T dt + \int_0^T \cos 2r\omega t\,dt\right]$$
$$= \frac{A_r}{2}.T \quad \left[\because \int_0^T \cos 2r\omega t\,dt = 0\right]$$
$$\therefore A_r = \frac{2}{T}\int_0^T y\cos r\omega t\,dt \qquad \text{——— (3)}$$

Evaluation of B_r :— Multiplying equation (1) by $\sin r\omega t\,dt$ and integrating from 0 To T and proceeding as above, we get

$$B_r = \frac{2}{T}\int_0^T y\sin r\omega t\,dt \qquad \text{——— (4)}$$

The value of the Coefficient A_0, A_r and B_r Can be used to analyse a given wave-form

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