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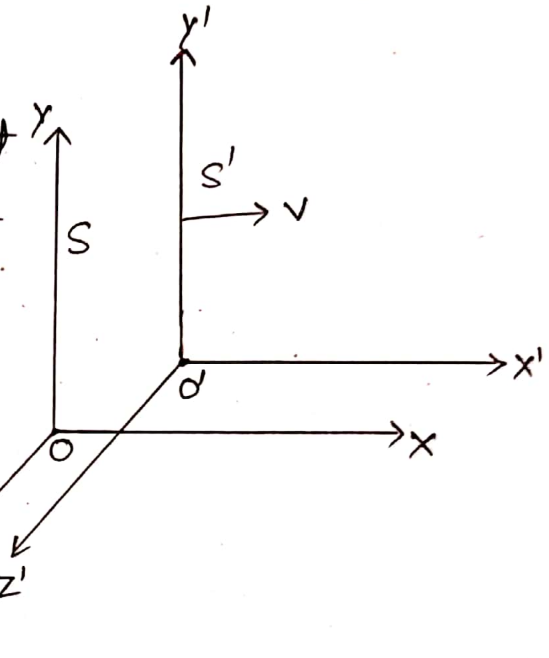
B. Sc (Part-I) (Hons)

TOPIC:- Lorentz Transformation equation:-

A set of transformation equation have been ~~def~~ developed directly from the postulates of special theory of relativity.

Let us assume that a measurement in the x-direction made in s is proportional to that made in s'.



$$\therefore \; x' \propto (x - vt)$$

$$\text{or,} \quad x' = \gamma (x - vt) \quad \text{——— (1)}$$

According to postulate 1, the laws of physics must have the same from in both s and s'. Therefore the corresponding equation of x in terms of x' and t' will be of the same form as equation (1) except that v will be replaced by -v

$$\therefore \; x = \gamma (x' + vt') \quad \text{——— (2)}$$

However, the time Co-ordinates t and t' are not equal. Let us substitute the value of x' from equation (1) in equation (2)

$$x = \gamma[\gamma(x - vt) + vt']$$

or, $$x = \gamma[\gamma x - \gamma vt + vt']$$

or, $$\frac{x}{\gamma} = \gamma x - \gamma vt + vt'$$

or, $$vt' = \frac{x}{\gamma} - \gamma x + \gamma vt$$

$$\therefore\ t' = \frac{x}{\gamma v} - \frac{x\gamma}{v} + \gamma t$$

$$t' = \gamma t - \frac{\gamma x}{v}\left(1 - \frac{1}{\gamma^2}\right) \quad \text{—— (3)}$$

Similarly,

$$t = \gamma t' + \frac{\gamma x'}{v}\left(1 - \frac{1}{\gamma^2}\right) \quad \text{—— (4)}$$

Now, γ can be evaluted from the postulate II. Let us suppose that a light signal is given at the origin O at time $t=0$ and $t'=0$. This means O and O' Coincide. The signal travels with a speed c which is same for both the frames as stated in postulate II. Its position as seen from S and S' after some time is given by

$$x = ct \quad \text{and} \quad x' = ct'$$

Substituting for x and x' in equation (1) and (2), we get

$$ct' = \gamma(ct - vt)$$

$$\therefore\ \left.\begin{aligned} ct' &= \gamma t(c - v) \\ \text{and} \quad ct &= \gamma t'(c + v) \end{aligned}\right\} \text{—— (5)}$$

Multiply these two equations

$$c^2 t t' = \gamma^2 t t' c^2 (1 - v^2/c^2)$$

$$\therefore \gamma^2 = \frac{c^2 t t'}{t t' c^2 (1 - v^2/c^2)}$$

$$\therefore \gamma^2 = \frac{1}{1 - v^2/c^2}$$

$$\therefore \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{——— (6)}$$

Substituting this value in equation (1) and (2), we get

$$\left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ \text{and } x &= \frac{x' - vt'}{\sqrt{1 - v^2/c^2}} \end{aligned}\right\} \quad \text{——— (7)}$$

Again from equation (6)

$$\gamma^2 = \frac{1}{1 - v^2/c^2}$$

$$1 - v^2/c^2 = \frac{1}{\gamma^2}$$

$$\text{or, } 1 - \frac{1}{\gamma^2} = v^2/c^2$$

$\therefore$ From equation (3)

$$t' = \gamma t - \frac{\gamma x}{v} \left(v^2/c^2 \right)$$

$$= \gamma \left(t - \frac{vx}{c^2} \right)$$

$$\text{or, } t' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t - \frac{vx}{c^2} \right)$$

$$\text{or, } t' = \frac{(t - xv/c^2)}{\sqrt{1 - v^2/c^2}}$$

and from equation (4)

$$t = \frac{(t' + x'v/c^2)}{\sqrt{1 - v^2/c^2}} \quad \text{—— (8)}$$

In of general when the light signal is not restricted to travel along the x-axis alone, we get two more relation

$$\left.\begin{aligned} y' &= y \\ \text{and } z' &= z \end{aligned}\right\} \quad \text{—— (9)}$$

Since these distance are measured perpendicular to v.

Equation (7), (8) and (9) are known as Lorentz transformation equation. They are used for the transformation of the space and time co-ordinates of an event from one frame of reference into the other in uniform relative motion.