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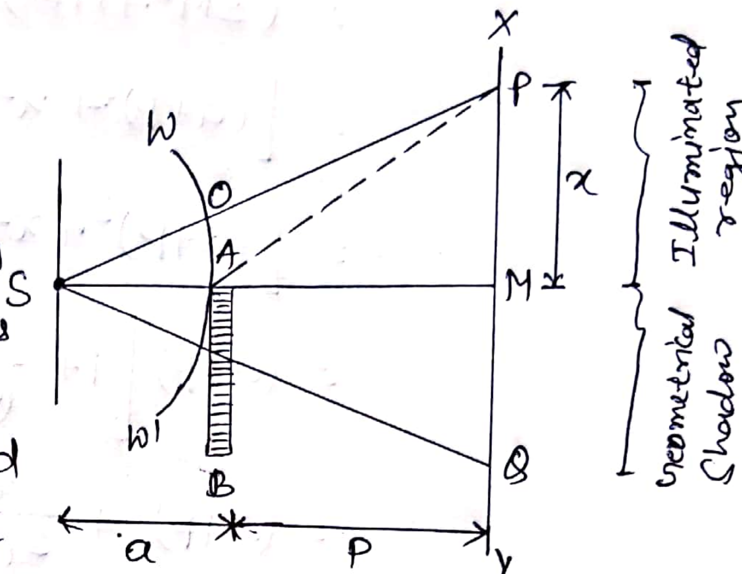
B. Sc (Part-II) Hons

Paper-III

Mathematical theory of the formation of fringes due to diffraction at a straight edge



Let us join PA.
Now as seen ~~abo~~ P is a maximum or minimum according as AO contains and odd or even number of half period strips, i.e. according as the path difference (AP−OP) is equal to an odd or even number of half-wavelengths.

Thus

$$AP - OP = (2n+1)\frac{\lambda}{2} \quad \text{(for maximum)} \quad \text{—(1)}$$

and

$$AP - OP = 2n\frac{\lambda}{2} \quad \text{(for minima)} \quad \text{—(2)}$$

where $n = 0, 1, 2, 3, \ldots$

putting $SA = a$, $AM = p$ and $MP = x$, we have

$$AP = (AM^2 + MP^2)^{1/2}$$

$$= (p^2 + x^2)^{1/2}$$

$$= p\left(1 + \frac{x^2}{p^2}\right)^{1/2}$$

$$= p\left(1 + \frac{1}{2}\frac{x^2}{p^2} + \cdots\right)$$

$$\therefore AP = p + \frac{x^2}{2p} \quad (\text{approx})$$

and

$$OP = SP - SO$$

$$= SP - SA$$

$$= (SM^2 + MP^2)^{1/2} - SA$$

$$= \left[(a+p)^2 + x^2\right]^{1/2} - SA$$

$$= \left[(a+p)^2 + x^2\right]^{1/2} - a$$

$$= (a+p)\left[1 + \frac{x^2}{(a+p)^2}\right]^{1/2} - a$$

$$= (a+p) + \frac{x^2}{2(a+p)} - a$$

$$= p + \frac{x^2}{2(a+p)}$$

$$\therefore AP - OP = \left(p + \frac{x^2}{2p}\right) - \left(p + \frac{x^2}{2(a+p)}\right)$$

$$= \frac{x^2}{2p} - \frac{x^2}{2(a+p)}$$

$$= \frac{ax^2}{2p(a+p)}$$

Substituting the value of $(AP - OP)$ in eqn (1) we have the condition for maxima

$$\frac{ax^2}{2p(a+p)} = (2n+1)\frac{\lambda}{2}$$

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or $\quad x = \sqrt{\frac{P(a+P)}{a}\left\{\frac{(2n+1)\lambda}{2}\right\}}$

or, $\quad x = K\sqrt{2n+1}$

where K is a Constant and $n = 0, 1, 2, ---$

Thus the distance between Successive maxima for M are (putting $n = 0, 1, 2, 3, ---$ in the above equation

$$x_1 = K$$
$$x_2 = K\sqrt{3}$$
$$x_3 = K\sqrt{5}$$
$$x_4 = K\sqrt{7}$$

The Separation between Successive maxima are

$$x_2 - x_1 = K(\sqrt{3}-1) = 0.73K$$
$$x_3 - x_2 = K(\sqrt{5}-\sqrt{3}) = 0.50K$$
$$x_4 - x_3 = K(\sqrt{7}-\sqrt{5}) = 0.43K$$

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which are clearly decreasing. Similarly, the separation between Successive minima are decreasing. Thus as we move M on the Screen, the fringes Come closer and closer.