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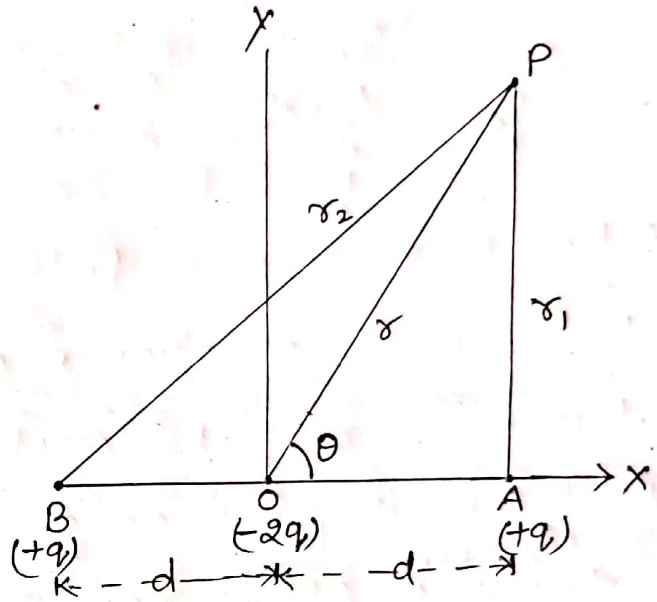
B.Sc (Part- II) Hons

TOPIC :— potential due to an electric quadrupole :—



Let us Consider a linear quadrupole AB which Contain (+q) charge at A and B, and (-2q) Charge at its centre O. Let us that the potential is to be Calculated at a point P where OP makes an angle θ with AB, AP and BP are joined. Let $OP = r$, $AP = r_1$ and $BP = r_2$

$\because$ potential at P due to charge A is

$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1}$$

potential at P due to charge at B is

$$V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_2}$$

potential at P due to charge at O is

$$V_3 = \frac{1}{4\pi\epsilon_0} \cdot \frac{-2q}{r}$$

$\therefore$ potential at p due to whole quadrupole is

$$V = V_1 + V_2 + V_3$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right)$$

$$\therefore V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right) \quad \text{——} \; (1)$$

$\because OA = OB = d$

$\therefore$ From ΔOAP

$$r_1^2 = r^2 + d^2 - 2rd\cos\theta$$

$$\therefore r_1 = \sqrt{r^2 + d^2 - 2rd\cos\theta}$$

$$r_1 = \left[r^2 + d^2 - 2rd\cos\theta\right]^{1/2}$$

$$r_1 = \left[r^2\left(1 + \frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)\right]^{1/2}$$

$$r_1 = r\left[1 + \frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right]^{1/2}$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left[1 + \left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)\right]^{-1/2}$$

$$\because \left[(1-x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8}\right]$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right) + \frac{3}{8}\cdot\frac{4d^2}{r^2}\cos^2\theta + \cdots\cdots\right]$$

$\because r >> d$, $\therefore$ higher power terms are neglected.

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{d^2}{2r^2}(3\cos^2\theta - 1) + \frac{d}{r}\cos\theta\right]$$

Similarly

$$\frac{1}{r_2} = \frac{1}{r}\left[1 + \frac{d^2}{2r^2}(3\cos^2\theta - 1) - \frac{d}{r}\cos\theta\right]$$

from eqn ①

$$V = \frac{q d^2}{4\pi \epsilon_0 r^3}(3\cos^2\theta - 1)$$

$$= \frac{2 d^2 q}{8\pi \epsilon_0 r^3}(3\cos^2\theta - 1)$$

The quantity $2qd^2$ is called the electric quadrupole moment denoted by P

$$\therefore V = \frac{P}{8\pi \epsilon_0 r^3}(3\cos^2\theta - 1)$$

This is required equation

Thus the potential due to linear quadrupole varies inversely as the cube of the distance.