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B.Sc (part-II) Hons
paper-III

Relation between spontaneous and Stimulated Emission probabilities.

Consider an assembly of atoms in thermal equilibrium at temprature T with radiation of frequency ν and energy density $u(\nu)$. Let N_1 and N_2 be the number of atoms in State 1 ~~that~~ and 2 respectively at any instant. The number of atoms in state 1 that absorb a photon and rise to state 2 per unit time is

$$N_1 P_{1\to 2} = N_2 B_{12} u(\nu)$$

Also the number of atoms in state 2 that drop to 1, either spontaneously or under stimulation, emiting a photon per unit time is

$$N_2 P_{2\to 1} \; N_2 [A_{21} + B_{21} u(\nu)]$$

For equilibrium, the absorption and emission must equally. Thus

$$N_1 P_{1\to 2} = N_2 P_{2\to 1}$$

$$\text{or,} \quad N_2 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$$

$$\text{or,} \quad u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$\text{or,} \quad u(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{N_1}{N_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

Einstein proved thermodynamically that probability of absorption must be equal to the probability of Stimulated emission. That is $B_{12} = B_{21}$

So that

$$u(\nu) = \frac{A_{21}/B_{21}}{\frac{N_1}{N_2} - 1}$$

The equilibrium distribution of atoms among different energy states is given by Baltzmann's law as.

$$\frac{N_1}{N_2} = e^{(E_2-E_1)/KT} = e^{h\nu/KT}$$

When $h\nu/KT \ll 1$ then $e^{h\nu/KT} \simeq 1 + \frac{h\nu}{KT}$

So that $u(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{KT}{h\nu}$

Comparing it with Rayleigh-Jeans formula

$u(\nu)d\nu = \frac{8\pi KT\nu^2}{C^3} d\nu$, we get the ratio between the spontaneous emission and induced emission co-efficients as

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3}$$

putting this value in the above formula, we get the plank's formula as

$$u(\nu) = \frac{A_{21}/B_{21}}{e^{h\nu/KT} - 1} = \frac{8\pi h\nu^3}{C^3} \cdot \frac{1}{e^{h\nu/KT} - 1}$$

This is clear that this ratio is proportional to ν^3 which means that the probability of spontaneous emission increases rapidly with energy difference between two states.

If a system (with $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3}$) is subjected to a radiation field at f, the stimulated emission can exceed the spontaneous emission if

$$B_{21}u(\nu) > A_{21} \quad \text{or,} \quad u(\nu) > \frac{A_{21}}{B_{21}}$$

The power absorbed during absorption will be

$$P_{abc} = B_{12}u(\nu)(N_1 - N_2)h\nu$$

because, in general case, $N_1 > N_2$

If N_2 were made large than N_1 by some artificial means, there will be emission of power instead of absorption.