prof. Vidya Sagar
Department of Physics
B.Sc (Part-1)(Hons)

TOPIC:- Relativistic formula for the addition of velocities

Let there are two frames of reference S and S', in which S' is moving with a Constant velocity v relative to S in x-direction. Let a body moves a distance dx in time dt in S.

If u be the velocity of the body measured by an observer in S, then $u = \frac{dx}{dt}$. For an observer in S' the distance and time will appear as dx' and dt'. Hence for him the velocity of the body will be

$$V = \frac{dx'}{dt'}$$

From the Lorntz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \quad dx' = \frac{dx - v\,dt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad dt' = \frac{dt - v\,dx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \quad V = \frac{dx'}{dt'}$$

$$= \frac{dx - v\,dt}{dt - v\,dx/c^2}$$

$$= \frac{\frac{dx}{dt} - v}{1 - v \cdot \frac{dx}{dt}/c^2}$$

$$\therefore v = \frac{u - v}{1 - uv/c^2}$$

and

$$u = \frac{u + v}{1 + uv/c^2}$$

If $v = c$ then

$$u = \frac{u + c}{1 + uc/c^2}$$

$$u = \frac{u + c}{1 + u/c}$$

$$\therefore u = \frac{u + c}{\frac{c + u}{c}}$$

$$\therefore u = c$$

Thus, both observer measure the Same value of the speed of light. The velocity of light is absolute independt of the frame of reference. This is one of the postulates of relativity.