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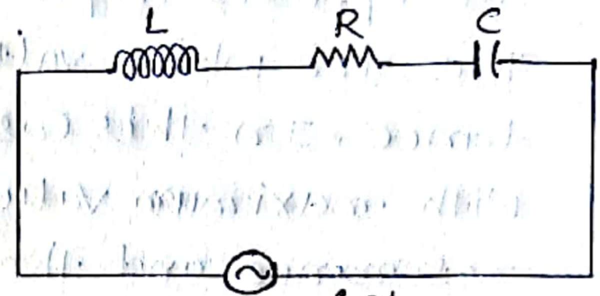
B.Sc (Part-II) Hons
Paper-IV

Resonance in L.C.R. Series Circuit.



$E = E_0 e^{j\omega t}$

Let us consider a circuit containing an inductance L, a resistance R, and a capacitance C in series and an alternating e.m.f. $E = E_0 e^{j\omega t}$ is applied to it as shown in figure. The impedence $\vec{Z}$ of the combination is equal to the sum of the impedences of three parts —

(i) Impedance due to resistance $R = R$

(ii) Impedance due to inductance $L = j\omega L$

(iii) Impedance due to capacitance $C = \frac{1}{j\omega C}$

$$\therefore Z = R + j\omega L + \frac{1}{j\omega C} =$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$\therefore$ The magnitude of this impedance is

$$|\vec{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$\therefore$ The current in the circuit is

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

If the frequency of applied voltage is varied, so that

$$\omega L - \frac{1}{\omega C} = 0$$

then the impedance Z is minimum and reduces to $Z = R$. In this case the applied e.m.f is in phase with the current. The P.D. across the inductance and capacitance are equal in magnitude but opposite in phase and therefore cancel each other. Thus, the whole voltage is dropped across the resistance. In this case the current is purely resistance with maximum value. This is called the electrical resonance and the circuit L-C-R is called Series-resonant circuit.

Hence, at resonance

$$\omega L = \frac{1}{\omega C}$$

$$\text{or, } 2\pi f L = \frac{1}{2\pi f C}$$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

where f is the frequency of the impressed e.m.f. But $\frac{1}{2\pi\sqrt{LC}}$ is the natural frequency of the circuit when the resistance in the circuit is zero. These, at resonance the frequency of the impressed e.m.f. should be equal to the natural frequency of the circuit when the resistance of the circuit is zero. Under this condition the series circuit allow the maximum current $I_0 = \frac{E_0}{R}$ to flow through it, and this current is in phase with the applied e.m.f. At resonance the frequency of the applied e.m.f is called the Resonant Frequency denoted by f_R

$$\therefore f_R = \frac{1}{2\pi\sqrt{LC}}$$

This shows that the resonant frequency depends on the product of L and C and does not depend on R.