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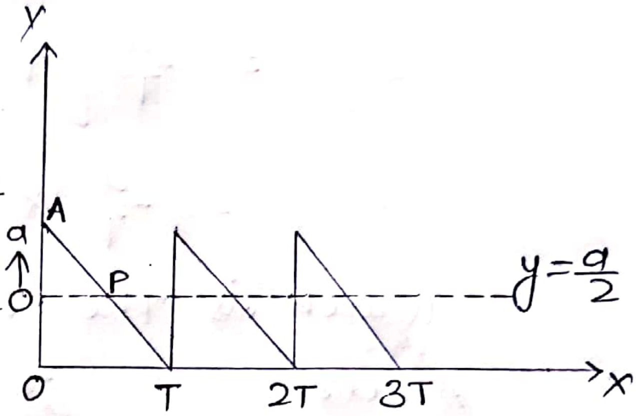
B.Sc (Part-1) Hons
Paper-1

Saw-Tooth Wave



A Complex vibration of Saw-tooth wave in which the displacement is linear and is given by

$$y = a \text{ at } t = 0$$

and $y = 0$ at $t = T$

Let the displacement at t is y.

$\therefore$ From the Similar triangles OAT and PA, we have

$$\frac{y}{a} = \frac{T-t}{T} = 1 - \frac{t}{T}$$

$$y = a\left(1 - \frac{t}{T}\right)$$

Here, the time axis passes through the lowest points of the displacement Curve. The Fourier's Series is

$$y = f(\omega t) = A_0 + A_1 \cos\omega t + A_2 \cos 2\omega t + \cdots + A_r \cos r\omega t + \cdots + B_1 \sin\omega t + B_2 \sin 2\omega t + \cdots + B_r \sin r\omega t + \cdots$$

where

$$A_0 = \frac{1}{T}\int_0^T y\, dt, \quad A_r = \frac{2}{T}\int_0^T y \cos r\omega t\, dt$$

$$\text{and } B_r = \frac{2}{T}\int_0^T y \sin r\omega t\, dt$$

$\because y = a$ at $t=0$ and $y=0$ at $t=T$

$$\therefore A_0 = \frac{1}{T}\int_0^T a\left(1-\frac{t}{T}\right)dt$$

$$= \frac{a}{T}\left[\int_0^T dt - \frac{1}{T}\int_0^T t\,dt\right]$$

$$= \frac{a}{T}\left[T - \frac{T}{2}\right]$$

$$= \frac{a}{T} \times \frac{T}{2} = \frac{a}{2}$$

which is the ordinate of the axis of curve

Again

$$A_r = \frac{2}{T}\int_0^T y \cos r\omega t\,dt$$

$$= \frac{2}{T}\int_0^T a\left(1-\frac{t}{T}\right)\cos r\omega t\,dt = 0$$

(for all value of r)

Now

$$B_r = \frac{2}{T}\int_0^T y \sin r\omega t\,dt$$

$$= \frac{2}{T}\int_0^T a\left(1-\frac{t}{T}\right)\sin r\omega t\,dt$$

$$= \frac{2a}{T}\int_0^T \sin r\omega t\,dt - \frac{2a}{T^2}\int_0^T t \sin r\omega t\,dt$$

$$\because \int_0^T \sin r\omega t = 0$$

$$\therefore B_r = -\frac{2a}{T^2}\int_0^T t \sin r\omega t\,dt$$

on integration by parts we have

$$B_r = \frac{2a}{T^2}\left[\left\{t\frac{\cos r\omega t}{r\omega}\right\}_0^T - \int_0^T \frac{\cos r\omega t}{r\omega}.dt\right]$$

$$= \frac{2a}{T^2}\left[\frac{T}{r\omega}\cos 2\pi r - \left\{\frac{\sin r\omega t}{r^2\omega^2}\right\}_0^T\right]$$

$$= \frac{2a}{T^2}\left[\frac{T \times T}{r \times 2\pi} \times 1 - \left\{\frac{\sin 2\pi r}{r^2\omega^2} - \frac{\sin 0}{r^2\omega^2}\right\}\right]$$

$$\text{or,} \quad B_r = \frac{2a}{T^2}\left[\frac{T^2}{2\pi r} - 0\right]$$

$$B_r = \frac{2a}{T^2} \times \frac{T^2}{2\pi r}$$

$$\therefore \quad B_r = \frac{a}{\pi r}$$

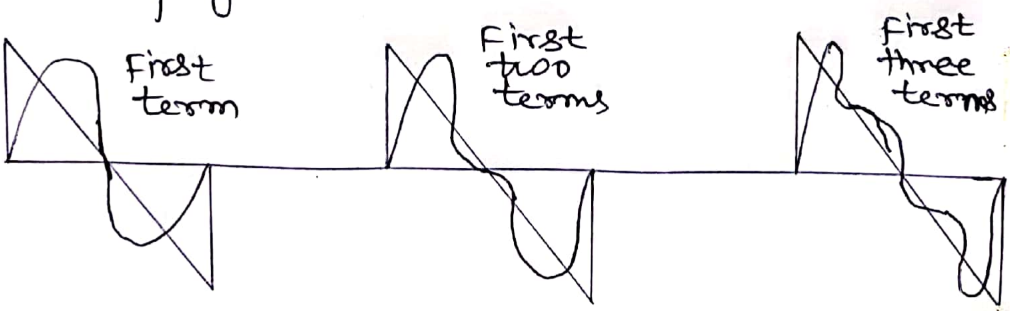
$\therefore$ From the Fourier's Series, we have

$$y = \frac{a}{2} + \frac{a}{\pi}\sin\omega t + \frac{a}{2\pi}\sin 2\omega t + \frac{a}{3\pi}\sin 3\omega t + \cdots$$

Thus the given Complex vibration has the axis $y = \frac{a}{2}$ above time axis and the various Components are

$$\frac{a}{\pi}\sin\omega t, \; \frac{a}{2\pi}\sin 2\omega t, \; \frac{a}{3\pi}\sin 3\omega t, \cdots$$

The Frequencies are in ratio 1:2:3 ---- and the amplitudes are in the ratio $1 : \frac{1}{2} : \frac{1}{3} : \cdots$ and so on. The addition of Successive terms of the Series is indicated graphically. It is Seen that greater the number of terms used the closer the resemblance between resultant Curve and the Curve under analysis.



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