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B.Sc (part-I) Hons

- Surface Tension and Surface Energy.
 - Expression for excess pressure over a curved liquid surface.
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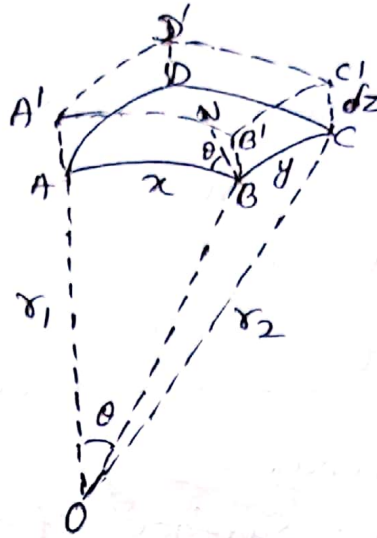
The surface of a liquid tends to contract and occupy the minimum possible area. This property is called its surface tension. It is measured by the force per unit length of a line drawn in the surface of the liquid, acting perpendicularly to it and tangentially to the surface of the liquid and tending to pull the surface apart along the line. Its unit is N/m .

Any strained body possesses potential energy which is equal to the work done in placing it in the present state. The surface of a liquid has also a strained system and hence the surface of a liquid has also a potential energy which is equal to the work done in creating the surface.

This energy per unit area of the surface of a liquid is called its surface energy. Its unit is J/m^2 .

Theory of Surface ~~energy~~ tension and surface energy:— The surface tension and surface energy of a liquid arise out of inter attraction of its molecules and a fundamental principle that system seeks a state of minimum potential energy.

Expression for excess pressure over a Curved liquid surface:-



Suppose ABCD is an element of a curved membrane having principal radii of curvature r_1 and r_2 . Let p be the excess pressure on the concave side of the element. Further let x and y be the sides of the element.

Let us a small normal displacement of the element by dz so that it reaches the new position A'B'C'D' having sides $x+dx$ and $y+dy$. The work done by the pressure is $(pny) \cdot dz$

$$\begin{aligned} \text{The increase in Area} &= (x+dx)(y+dy) - xy \\ &= xy + xdy + ydx + dx \cdot dy - xy \\ &= xdy + ydx \end{aligned}$$

neglecting the small product of $dx \cdot dy$.

$$\text{Increase in Surface energy} = 2s(xdx + ydy)$$

It is multiplied by 2 because there are two surface. The work done is equal to the increase in surface energy of the membrane.

$$(pxy)dz = 2S(xdy + ydx)$$

$$\text{or } p = 2S \left(\frac{1}{y} \cdot \frac{dy}{dz} + \frac{1}{x} \cdot \frac{dx}{dz} \right)$$

Let the normal to the membrane at A and B meet at O. Then O is the centre of curvature and $OA = OB = r_1$ = the first principal radius of the membrane. Draw a line BN parallel to AA' then $\angle AOB = \angle NBB' = \theta$

$$\therefore \theta = \frac{x}{r_1} \text{ and also } \theta = \frac{dx}{dz}$$

$$\therefore \frac{x}{r_1} = \frac{dx}{dz}$$

$$\text{or } \frac{1}{r_1} = \frac{1}{x} \cdot \frac{dx}{dz}$$

$$\text{Similarly } \frac{1}{r_2} = \frac{1}{y} \cdot \frac{dy}{dz}$$

$$\therefore \boxed{p = 2S \left(\frac{1}{r_1} + \frac{1}{r_2} \right)}$$

This expression is true also for the excess of pressure inside a liquid bubble. If there is only one surface, as for example, in the case of a liquid drop, or air bubble in a liquid

$$p = S \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

For a soap bubble $r_1 = r_2 = r$

$$\therefore \boxed{p = \frac{4S}{r}}$$

For an air bubble in a liquid

$$\boxed{p = \frac{2S}{r}}$$